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Variations on Perfection: The Sequin-Collins Sculpture Generator

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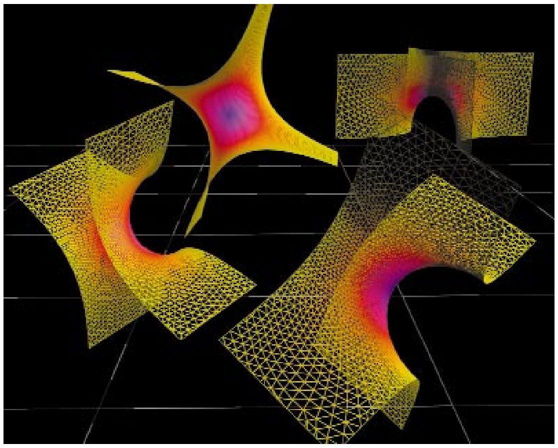
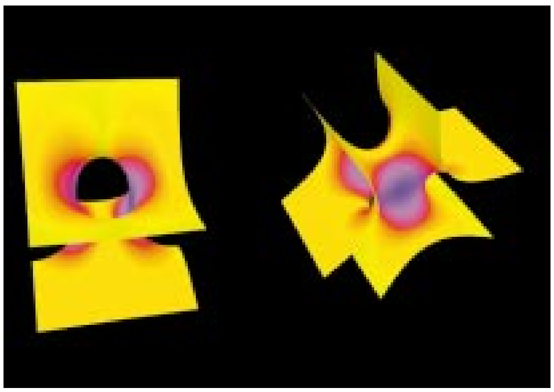
Rarely do the pursuits of fine artists, computer scientists/engineers, and mathematicians converge. Sculpture inspired by "minimal surfaces" offers an exception. As the term implies, a minimal surface is concerned with economy, both in surface area and in the energy expended to bend the surface. Such surfaces can extend infinitely and do not self-intersect. For example, Scherk's Second Minimal Surface is characterized by interlocking saddle forms set 90 degrees to each other (see Figure 1). The number of saddle forms ("orders") can vary. (See the Mathematical Sciences Research Institute's Scientific Graphics Project for a survey of surface descriptions: www.msri.org/publications/SGP/index.html.)



1 Scherk's Second Minimal Surface of the second order, in wireframe. The single surface creates two interlocking saddle forms.



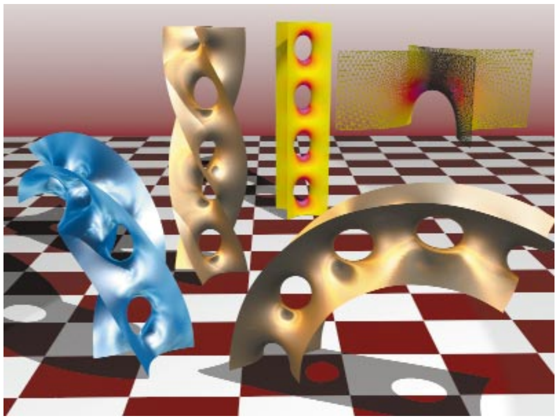
2 A Scherk's surface of the third order—three saddle forms coming together.

Figure 2 shows a Scherk's surface of the third order (three saddles coming together). Stacking and mirroring these surfaces creates an interlocking surface comprised of saddle surfaces and holes (see Figure 3), and extending the stack results in a form known as a Scherk's Tower. Figure 3 shows examples of a Scherk's Tower of the second order in its generic form and with a twist, a bend, and a twist plus a bend applied.

A peculiar pursuit

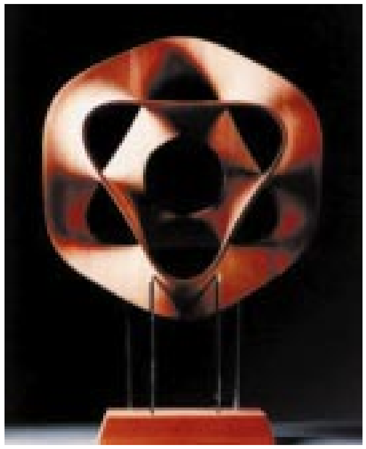
Artists, collectors, and critics increasingly accept digital tools in the hands of artists. We accept software tools for mathematicians. We expect software developers to engineer specialized tools because generally the same software doesn't serve the ends of both artists and mathematicians. The fine artist is driven by aesthetics, the mathematician by theory and structure, and the computer scientist/engineer by expanding the visualization capability. Software rarely bridges these fields.

What about sculpture based on minimal surfaces? That genre creates a strange ripple in 20th century art history and theory: source

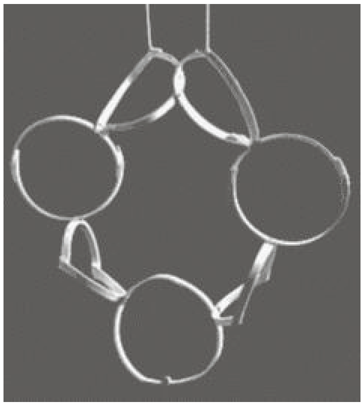


3 Mirroring and stacking the minimal surface creates a Scherk's Tower. Here you see examples of the minimal surface (in wireframe), the basic tower, and that same tower with texture mapping and a bend, a twist, and a twist plus a bend applied.

4 Collins' early sculptures concerned minimal surfaces and saddle forms, but were developed intuitively, without regard to the underlying mathematics



5 Collins' "Hyperbolic Hexagon" (1995, wood, 20 by 20 by 8 inches).



6 The initial armature and beeswax-PVC pipe models from which Collins constructed the "Hyperbolic Heptagon."

material for abstract sculpture usually derives from forms in nature (including the artist's psyche), as evidenced by pre-World War II European sculpture, or pure mathematics, as expressed in formalist American minimalist sculpture. While most artists don't analyze themselves or their work in these terms, their output usually favors one of these sources, not both. Artists investigating minimal surfaces are an exception—such surfaces, although best described mathematically, also exist in nature, for example as the surface made by soap film spanning two edges.

Nonetheless, these disparate elements of geometry, aesthetic vision, and engineering come together when you can get mathematics quickly and efficiently as geometry on graphics workstations, interactively, and in real time. Specifically, Carlo Sequin's Scherk-Collins Sculpture Generator brings the three disciplines together, generating variations on Scherk's second minimal surface at better than 10 frames per second to facilitate the sculptor's own design visualization process.

The sculptor

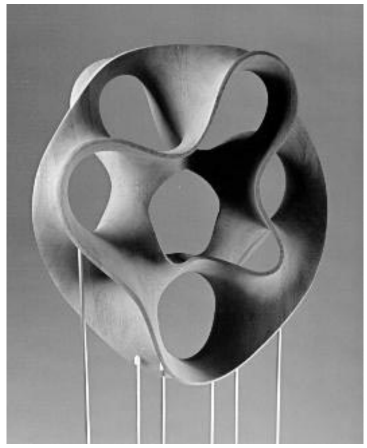
Beginning in the 1980s, Brent Collins produced a series of wood sculptures dealing with the convex exteriors of enclosing ellipsoids, with curvature spanning from the outside edges through a hollow interior—in other words, stacked saddle forms (see Figures 1 and 2). But Collins' work in his studio in Gower, Missouri evolved from artistic vision and intuition, not mathematics, even though it increasingly concerned the minimal nature of surfaces.

In 1989, Collins learned that mathematicians knew this family of surfaces as Scherk's Second Minimal Surface. (Technically, Collins' works resembled Scherk's Towers, but hemispherically capped to achieve an external elliptical shape and interwoven to produce seamless holes; see Figure 4). Several years later, when viewing computer graphics of geometric forms, he discovered his works were based on truncations of Scherk's surfaces and the genus-two Costa surfaces (regarding the rounded, bracketed ends). A physicist first suggested that his surfaces reflected the economy of a soap film's surface. Through his intuition and artistry he'd arrived at a union of nature and mathematics.

In late 1994, Collins bent and twisted a Scherk's Tower of the second order into a circular, or toroidal, closed form. This produced the "Hyperbolic Hexagon" (see Figure 5). First, he had to choose the form. This required building a maquette (an accurate physical model, often at reduced scale) of the proposed piece, which usually takes three weeks. For the "Hyperbolic Hexagon" he used beeswax and PVC pipe to build the armature and prototype (see Figure 6). Then he took precise measurements of cross-sections of the piece. These were applied to one-inch thick boards, which he then cut, stacked, and glued together to form a rough first shape. Finally, he finished and refined the surfaces by hand, a labor-intensive process usually requiring three months.

This intuitive technique suffers from two limitations: the complexity of the form that can be prototyped, and the time it takes to evaluate a worthy candidate. If there were a way to visualize and evaluate the myriad of potential prototypes interactively in CG space, and from the chosen model generate usable measurements and blueprints, the design process might overcome these limitations and enable building more sophisticated and complex forms.

7 **The virtual Hexagon generated by the Sequin program.**



8 **Collins' "Hyperbolic Heptagon" (finished).**

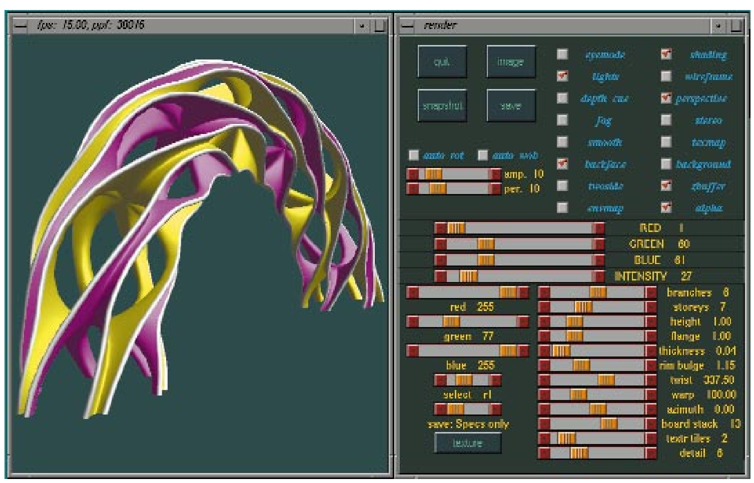
9 **The virtual Heptagon generated by the Sequin program.**

The researcher

Carlo H. Sequin joined the University of California at Berkeley as a professor of electrical engineering and computer sciences in 1977. He has concentrated on computer graphics and geometric modeling since the early 1980s, when he built the Berkeley Unigrafix system from which he developed his first sculpture modeling program, "mkworm." It could sweep a shape along a straight-lined path and generate properly mitered corners. His research has continued over the past 15 years to provide a bridge between mathematics, computer visualization, and fine art sculpture. See http://http.cs.berkeley.edu/~sequin/.

The collaboration

Sequin and Collins came together through a mutual friend after Collins completed the "Hyperbolic Hexagon." In early 1995 Sequin suggested a prototyping program and proposed collaborating on a Hyperbolic Heptagon sculpture, a variation on the Hexagon, that would use an odd-number of Scherk's surfaces with an additional twist of 90 degrees. Collins developed the piece using his technique of building an armature of pipe and beeswax; Sequin continued to refine the program. Collins finished the piece in late 1995. Several months later Sequin sent him the first batch of graphic images from his generator program, including wood-texture-mapped renderings of the "Hyperbolic Hexagon" and "Hyperbolic Heptagon." (Figures 5 and 7 represent the real and virtual hexagon, and Figures 8 and 9 represent the real and virtual heptagon). The striking similarity between the real and virtual models demonstrated the feasibility of moving their future collaborative design efforts into the virtual environment.



10 **Screen shot of the Sculpture Generator I program interface and sample geometry.**

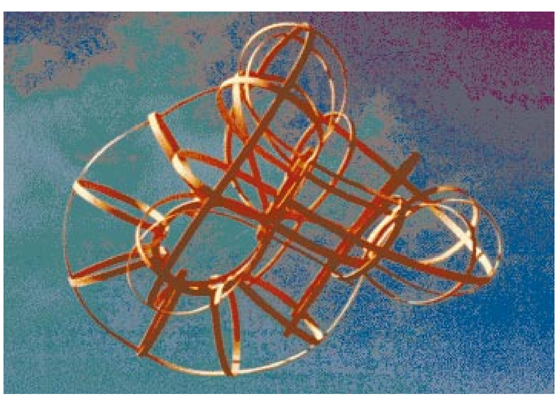
The Scherk-Collins Sculpture Generator is a C++ program (about 10,000 lines of code) running on a Silicon Graphics Infinite Reality 2 workstation with one processor. The interface and sample geometry appear in Figure 10. The name comes from the program's ability to generate a 3D model of the Scherk's surface with a variable number of orders and to twist and warp the object into shapes Collins uses in his sculptures. (It departs from Scherk's surface because it self-intersects; Sequin refers to this family of shapes as Scherk-Collins saddle rings.)

Working with the sculpture generator combines a search for inspiring forms with rapid elimination of unusable possibilities. More intriguing, it promotes collaboration between a computer scientist and engineer and a traditional fine artist in pursuing a shared aesthetic they approach from entirely different directions.

The program can generate surfaces from the first

11 **Stages in the fabrication of Collins' "Vox Solis" sculpture. The virtual form is sliced into cross-sections in the computer, with each slice mapped in a blueprint. One-inch wooden planes are cut according to the blueprints, laminated together, and finished by hand.**

12 **Photograph of an 11-inch rapid-prototype model of Collins' "Hyperbolic Heptagon," created using stereolithography.**



13 **Carlo Sequin's "Skeleton of a Boy's Surface" (16 inches, 1995).**

order (a single plane with a twist) to above the tenth order (an interwoven cable-like form). The surfaces ("flanges") can be elongated outward from center holes and made to any thickness. Flange edges can vary from flat to ellipsoidal. The tower shape can rotate along its axis, warp into a partial or complete torroidal shape, and twist along its axis. The twist varies with the orders (90 degrees for second order, 60 degrees for third, and so on) and is in multiples of the angle required to close the torroidal shape of an odd number of stories.

A user can map textures to a virtual sculpture, applying various colors or realistic materials to front, back, and edged surfaces; rotate the sculpture for viewing at any angle; and position it against different backgrounds. Since the program aims for real-time interactivity, users can control levels of detail and turn textures on and off as needed. The virtual model can be viewed as a single image or stereoscopically.

In the Irix version, the interface consists of about 12 sliders with approximately 100 settings per slider. Sequin ported the program to Windows NT as part of a collaborative exhibit with Collins in mid-1998. Because current NT hardware can't match SGI's Infinite Reality 2 in real-time rendering capability, the NT port did not include all the capabilities of the Irix version. Still, gallery attendees had the opportunity to work with the Scherk-Collins Sculpture Generator to better understand the design process and how the virtual model translates to its physical counterpart. (Nonetheless, Sequin has no immediate plans to make this software available to the public.)

Once Sequin and Collins settle on a final design, the program generates slice topology of the prototype in the form of life-size blueprints. Each sheet represents a one-inch slab of wood. A plotter using five different colors plots the location of sublayer topologies for each slab. Looking at these, Collins can deduce the angle and curves at each layer. He then cuts and shapes each one-inch board to match the drawing, finally laminating them together and finishing the assembled sculpture. (See Figure 11 for work-in-progress shots of "Vox Solis.")

The possibilities of rapid prototyping

Sequin presented his paper "Art, Math and Computers: New Ways of Creating Pleasing Shapes" at the Bridges Conference in Winfield, Kansas, July 1998. In it he views the traditional sculpture process as the interweaving of the two central components, *design* and *implementation,* which can go on simultaneously and alternately. Using the Sculpture Generator for the design phase separates these processes, restricting design entirely to the virtual environment and implementation to a fabrication environment. Also, it orders them—an unacceptable restriction for some artists.

Rapid prototyping technologies may someday provide the "implementation" counterpart to the Sculpture Generator "design" process. Figure 12 shows a photograph of an 11-inch stereolithographic model of Collins' "Hyperbolic Heptagon." The virtual model was designed in the Sculpture Generator. The data, in STL file format, guided a laser applied to an organic liquid photopolymer, layer by layer. The liquid hit by the laser solidifies; the process results in a solid maquette.

Currently, about a dozen different techniques use resins or various powders (nylon, paper, ceramic) to rapidly prototype objects directly from CG data. But the technologies remain in the infant stage in terms of cost and interactivity. For example, a rapid prototype model

Approaches Taken by Fine Artists

In a recent interview for *IEEE CG&A*, Carlo Sequin described three distinct approaches currently used by sculptors working at the junction between fine art sculpture, mathematics, and computers, referring to Helaman Ferguson, Bruce Beasley, and Brent Collins as examples of working styles.

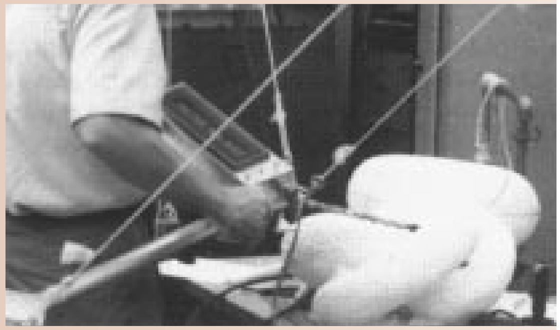
Helaman Ferguson possesses a unique ability to express his own mathematical formulations as abstract art. He generates his forms using mathematics and executes the sculpture with computer-assisted Stewart Suspension holding the drill (see Figure A). The suspension apparatus in effect frames real space to relate it to the computer's virtual space. The artist guiding the drill can drill sink-holes into the real stone and use the computer as a potentiometer to locate the analogous "virtual surface" inside (that is, the surface resulting from the mathematical expression as it would be placed within the stone; see Figure B).

For an in-depth view of his work, see *Helaman Ferguson: Mathematics in Stone and Bronze*, text by Claire Ferguson, published by Meridian Creative Group. (Find a review of this book at http://www.msstate.edu/Fineart_Online/Gallery/Ferguson/maths.html.)

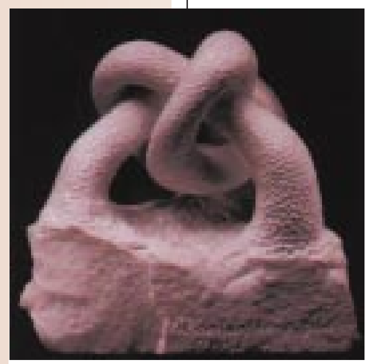
Bruce Beasley isn't concerned with minimal surfaces or the mathematics underlying them. He uses an off-the-shelf CAD package that provides primitive trapezoidal and pyramidal shapes. He arranges, pulls, and combines objects and portions of objects to arrive at a prototype form. Although the component shapes are primitive, they by no means collectively restrict themselves to the mathematical discipline at the root of Ferguson's or Collins' work. Beasley designs intuitively within virtual space, applying his artistry interactively through the software. Once he has arrived at a prototype form, he uses the software to generate plans. A foundry or fabricator caste the real-life bronze using those plans. Beasley then textures and finishes the final surfaces by hand in his studio. (See Figure C.)

Collins, of course, takes from each approach, using mathematics and software to arrive at the design prototype. He then executes the final sculpture entirely by hand.

A growing community of engineers, mathematicians, and artists (sculptors and non-sculptors) pursues the convergence of art and mathematics. In August 1998, UC Berkeley hosted the Art-Math 1998 Conference, consisting of three days of invited presentations followed by two days of intensive workshops. The second conference, ISAMA 99 (International Society of Art, Math, and Architecture), is scheduled for San Sebastian, Spain, 7-11 June 1999. Contact Nat Friedman, Dept. of Mathematics, University at Albany–SUNY, Albany, NY 12222, e-mail aartmath@math.albany.edu. For further information, including a program of the first conference, see http://http.cs.berkeley.edu/~sequin/AM98/index.html.



A The Ferguson-Stewart drill is computer aided to let the artist accurately locate the "virtual surface" within the real stone block.



B "Thurston's Hyperbolic Knotted Wye I," Helaman Ferguson (marble, 13 inches).

C Bruce Beasley's Intersections (cast bronze, 24 inches, 1993).

measuring 2.5 by 2.5 by 2.5 feet—one of the smaller Collins sculptures—could exceed $10,000 to create. Also, implementing sculpture with rapid prototyping is not interactive.

From my perspective, future development of rapid prototyping processes should try to reunite design and implemenation processes in the virtual environment, for example by combining the Sculpture Generator with an interactive technology. Using malleable materials to make prototypes (materials slow to harden or capable of being resoftened and molded) or light-based visualizations such as holograms would let artists interact directly with the medium during design and building of the prototype, just as in traditional sculpture. I'm not aware of any current research, development, or public discussion of these different approaches.

Sculpture Generator II

Sequin has been creating and showing his own mathematics-based artworks since 1985 (see Figure 13 and http://www.cs.berkeley.edu/~sequin/SCULPTS/sequin.html). This natural interest in artistry, his work with other artists, and his particularly successful collaboration with Collins have spurred the development of Sculpture

14 Charles O. Perry's "Eclipse" in the Hyatt Regency Hotel lobby, San Francisco (35 feet).

Generator II (SG II, a programming environment). Sculpture Generator I will be a subset of SG II. Sequin is working with Jordan Smith, a graduate student at Berkeley, to expand the range of shapes showing promise as abstract artworks.

When finished in about a year's time, SG II should have between 10 to 50 times the power of SG I. It will not be restricted to generating minimal surfaces. Also, SG II will contain a special Beads module: one bead will represent a one-rung ladder element that can be replicated along any prescribed path. This will permit shapes in the style of Charles Perry (see Figure 16). Another type of bead might implement a tire tread pattern, which would translate to a type of surface geometry.

The final SG II should support interactive design and manipulations at 10 frames per second on the Irix platform. Presently, the program is in the form of several C++ libraries. The current GUI is complicated by too many pop-up menus to be usable and is undergoing redesign. Sequin currently envisions a slider-driven interface similar to SGI's, but with three windows: one for designing beads, one for paths, and one for the Placement module and combined effect.

Possibilities using virtual reality

While on sabbatical in late 1996, Sequin worked at the University of North Carolina at Chapel Hill with graduate student Mark Mine to couple the sculpture generator with a VR interface. Wearing a head-mounted display, the viewer could interactively fly through the model, above and along the scaled-up surface, experiencing the form in a way not possible with any real-world counterpart.

The real-time and interactive VR possibilities of this technology suggest new directions and deserve development: If you can fly around and through an artwork, it elevates the importance of motion and the time dimension to equal or exceed dimensions governing form. To the extent virtual reality approximates the physical, questions arise regarding the rationale of converting the prototype sculpture into a physical artifact. This will become increasingly pertinent as haptic technologies mature.

For example, what does it mean if creation occurs in real time, interactively, in a virtual modeling environment where the artist discerns the one inspired model from millions of possibilities? What if the physical sculpture does not reflect or emphasize idiosyncrasies of the artist's hand in executing the piece? What is the import of the artist executing the sculpture by hand in a studio? Or, for that matter, why make the physical artifact at all if observers can fully experience the art within a virtual environment or represented in real space by a virtual model, hologram or otherwise?

Also, real-time interactive animation results in a unique artwork and experience separate and apart from any "snapshot model," just as the gestalt of a film differs from any single frame. This has three aspects: (1) the effect of moving around and through the sculpture, (2) the effect of interactive morphing, warping or otherwise transforming the form, and (3) the likelihood the interactive experience will not repeat in the same way.

Tools like Sequin's Sculpture Generator open a door to virtual artworks of a future time. But that's a different kind of artwork, in another medium. It's merely *different* from physical sculpture, not a replacement. To a species accustomed to interacting with a physical world, finding beauty and meaning in it, and memorializing it in the artistic record, the Sculpture Generator is practical and useful today. It may become indispensable tomorrow for designing ever more complex geometric artworks. To the extent it points to a new world of virtual art, even the act of creating tools like the Sculpture Generator becomes an art form in its own right. ■

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